Stochastic Optimal Control for Energy Internet: A Bottom-up Energy Management Approach

Haochen Hua, *Member, IEEE*, Yuchao Qin, Chuantong Hao and Junwei Cao, *Senior Member, IEEE*

Abstract—In this paper, an energy management issue is considered for energy Internet (EI) where microgrids (MGs) are interconnected via energy routers (ERs). Focusing on an individual MG, we propose controllers in micro-turbines (MTs) and the ER, such that the following three criterions are hold simultaneously. First, a bottom-up energy management approach is realized. Second, the operation cost of utilizing battery energy storage (BES) devices is minimized. Thirdly, the situation of over-control with respect to MTs is considered to be avoided. Besides, we develop a novel hybrid modelling method combining both recurrent neural networks (RNNs) and Ornstein-Uhlenbeck process to obtain accurate power models for both photovoltaic panels (PVs) and loads. Next, we formulate our energy management issue into a stochastic optimal control problem and solve it via dynamic programming approach. Finally, examples illustrating the feasibility of the proposed methods are provided.

***Index Terms*—Energy Internet; Microgrids; Neural Networks; Optimal Control; Stochastic Systems**

NOMENCLATURE

BES	Battery energy storage.
ER	Energy router.
EI	Energy Internet.
MT	Micro-turbine.
MG	Microgrid.
NN	Neural network.
PV	Photovoltaic panel.
RNN	Recurrent neural network.
SOC	State of charge.
P_{BES}^L	Upper limit for P_{BES}.
P_{PV}	Output power of PV.
P_{solar}	Theoretical solar radiation.
P_{PV}^D	Deterministic part of P_{PV}.
P_{PV}^S	Stochastic part of P_{PV}.
P_L	Power of loads.
P_L^D	Deterministic part of P_L.
P_L^S	Stochastic part of P_L.
P_{MT}	Output power of MT.
P_{BES}	Input/output power of BES.
P_{ER}	Input/output power of ER.
Q_s	Capacity of BES.
T_{MT}	Time constant of MTs.
η_{in}	Charging coefficient of BES.
η_{out}	Discharging coefficient of BES.

I. INTRODUCTION

Due to the gradually increasing cost of fossil fuels, accompanied with ecological issues and economic influences, more emphasis is put on utilizing renewable energy sources such as wind power generation, solar power generation and hydropower generation [1], [2]. In the future, when vast renewable energy sources are regarded as the backbone of the sustainable energy systems, an essential transformation and upgrading for the global energy infrastructure is urged [3], [4]. Based on smart grids focusing on the informatization and intellectualization of the existing power systems, recently, the new concept of EI is proposed as the 2.0 version of smart grid [5]-[7].

For a typical EI scenario, multiple MGs are interconnected, and electric power is dispatched from one MG to the others via ERs [7], [8]. The ERs, also known as energy hubs, or energy exchange devices, are viewed as power intermediary, such that power balance for the whole EI can be achieved [9], [10]. For each individual MG in an EI, the main power deviation on the power bus is expected to be regulated by its local BES devices and controllable MT output power with priority. If the local MG's power balance is difficult to be achieved autonomously, then wide area energy routing strategy shall be implemented by ERs, such that external power can be transmitted into/out of the local MG [7]-[10]. This is viewed as the bottom-up energy management approach for EI, which is different from the top-down mode in the existing power systems.

In the past decades, the energy management issues for MGs have received much attention and significant advances on this topic have been made; see, e.g., [11]-[15]. Efficient energy management strategies for a grid-tied residential MG is considered in [11]. A class of MG energy management problem is studied via the stochastic continuous time model in [12]. In [13], distributed control and optimization problems are studied for MGs. For applications of H_∞ control in an islanded MG, readers can refer to [14]. Coordinated energy management strategies for the networked MGs have been investigated in [15]. However, there has been few work considering problems of energy control and optimization in the field of EI, due to their complexity.

On the other hand, there exist many fundamental results focusing on prediction and modelling power of PVs and loads; see, e.g., [17]-[20]. However, the power modelling approaches adopted in some of the previous works have certain shortcomings. For example, in [17], semi-Markov model is used to describe PV power. Although such stochastic process is able to represent the randomness of PV power, the overall trend of PV output power is difficult to be acquired. In [18] and [20], NNs are used for the power modelling of loads and PVs, respectively. Although NNs are able to represent their complex patterns, the randomness in power dynamics of PVs and loads cannot be fully described. In [21], only stochastic differential equations driven by Wiener process are used to model the dynamics of the load power, without considering the technique of NNs.

In this paper, we consider designing control strategies such that the bottom-up energy management principle within an EI scenario is realized. Particularly, we focus on one single MG which is interconnected with the network of multi-MGs via an ER. The controllers are assumed to be set in MTs and the ER only, and our control targets mainly contain the following three aspects. Firstly, according to the bottom-up energy management principle, local power deviation shall be regulated by local BES devices and controllable MT output power with priority. Secondly, since some BES devices are relatively expensive and frequent large-scale charging/discharging would reduce their service life [16], the operation cost of utilizing BES devices shall be controlled to a relatively low level. Thirdly, the situation of over-control w.r.t. MTs and the ER shall be avoided, since strong controllers would involve additional operation cost and might destroy MTs and the ER [8].

Throughout the previous literatures with respect to (w.r.t.) EI, the above three criterions have *not* been considered *simultaneously*. To provide reliable solutions to these aforementioned challenges, a precise model for EI system is urged. In this article, some efforts are also put on developing a new modelling method regarding power of PVs and loads. Based on real data in [22], a class of novel hybrid model combining both RNNs [23] and Ornstein-Uhlenbeck process [24] is proposed for power modelling of PVs and loads. Next, we formulate the considered energy management issue for EI as a stochastic optimal control problem. Then, it is solved by dynamic programming approach provided by the open source C++ program BOCOPHJB [25]. Simulations show the effectiveness of the novel hybrid modelling method and the feasibility of the designed controller.

The importance and contribution of this paper can be outlined as follows.

1. With the proposed method, the bottom-up energy management principle for EI is achieved. Compared with previous works merely mentioning the concept of bottom-up, this is the first time that such key principle in EI is realized from the control perspective.

2. With such bottom-up principle, the supply and demand balance within each individual MG is maintained with priority, then power balance can be achieved in the whole EI scenario successfully. In this sense, the power supply for customers could be more reliable.

3. When designing the energy management controller, the rational utilization of BES devices is considered, such that the service life of BES devices can be prolonged effectively.

4. Under the proposed controller, the rational utilization of MTs is achieved, such that the situation of over-control is avoided.

5. It is notable that the aforementioned three targets have not been considered *simultaneously* in the field of EI. We claim that once the control strategy w.r.t. one individual MG is obtained, such control scheme can be applied to other MGs within the whole EI without essential difficulty.

6. The feasibility and effectiveness of our proposed controller are evaluated in the numerical simulations. The results demonstrate the advantages of the proposed method over the conventional approach (deterministic control scheme).

7. Apart from the main achievements in energy management issues, a novel modelling method combing both NNs and stochastic process are used to obtain precise power models for both PVs and loads. Particularly, the technique of using both RNNs and Ornstein-Uhlenbeck process to establish power models has *not* been considered before. It is highlighted that with such advanced power modelling approach, the results obtained for the energy management problem is more reliable.

The rest of the paper is organized as follows. Section Ⅱ describes the modelling for the dynamics of a MG within the scenario of EI. Section Ⅲ formula control problem and introduces the approach to solving it numerically. Section Ⅳ we provides so conclude our paper in Section .Ⅴ

II. SYSTEM MODELING

In this section, we introduce the components of the investigated MG within the scenario of EI. The power models for the MG components are formulated thereafter.

A. The MG Components and the Data Source

As is shown in Fig. 1, the MG studied in this paper is assumed to be composed of distributed PVs, loads, MTs and BES devices. We assume that it is connected to a multi-MGs network via an ER. The controllers are set in MTs and the ER only. Distributed PVs and loads are assumed to be uncontrollable. BES devices are considered to absorb the power oscillations on the power bus passively. In Fig. 1, the power deviation on the power bus can either be absorbed by BES devices or be eliminated by power input/output from the connected multi-MGs through the ER.

We use the full year data of power and weather in [22] to establish our new power models for PVs and loads. The power data include the electricity consumption and PV power generation of the civilian houses which joint the smart grid project in Austin, Texas, US. Both power consumption and generation are measured and recorded by smart meters at the minute level. The weather data are recorded hourly. The recorded weather information includes weather summary, temperature, humidity, atmosphere pressure, wind speed, cloud cover and probability of precipitation.

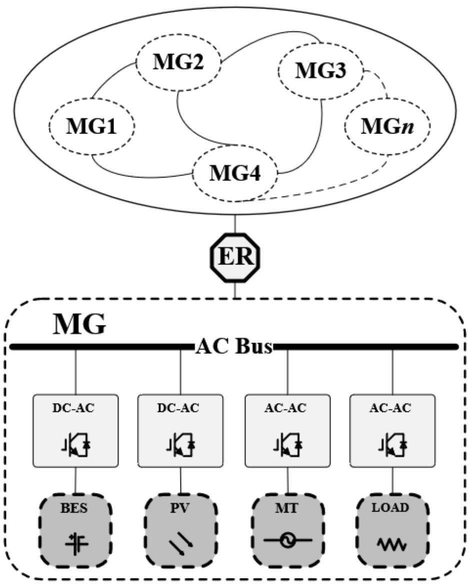

Fig. 1. Structure of the studied MG.

Based on the above data, the modelling of the considered MG is investigated at the minute level. Since the response time of BES devices is usually around several hundred microseconds or less [26], [27], we assume that the deviations on power bus can be absorbed by BES devices almost instantly. Thus, the regulation of BES power dynamics is not considered throughout this paper.

B. A Novel Hybrid Power Modelling Approach for PVs and Loads

In this article, we establish new power models for PVs and loads as a class of random variables containing both deterministic and stochastic terms. Mathematically, we denote such kind of random variables to be $X(t)$ which can be described as

$$X(t) = X^D(t) + X^S(t), \tag{1}$$

where $X^D(t)$ is the deterministic term and $X^S(t)$ is the stochastic term. In this paper, values of $X(t)$ represent for the real PV power and load power in [22].

1) Predictive model for $X^D(t)$

By using the square window function to smooth $X(t)$, we can eliminate the effect of $X^S(t)$ and obtain an approximated value of $X^D(t)$. Then, we are able to predict $X^D(t)$ by training NNs with relevant data in [22]. Long and short-term memory network (LSTM) is a kind of RNN and it has the ability to describe the relationship hidden behind time series [18]. Since the power of PVs and loads may be related to historical data, LSTM is adopted to predict $X^D(t)$. The structure of the NNs is shown in Fig. 2.

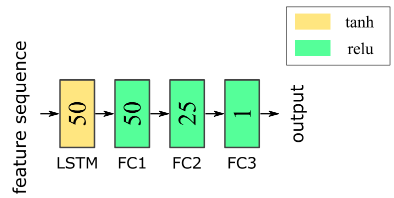

Fig. 2 Structure of the NN for prediction of $X_D(t)$.

As is shown in Fig. 2, the network is composed of a LSTM layer and three fully connected layers (denoted as FC1, FC2, FC3). There are 50 nodes in the LSTM layer, and the activation function in these nodes is hyperbolic tangent (denoted as 'tanh' in Fig. 2). The three fully connected layers contain 50, 25 and 1 nodes, respectively. The activation function for them is rectified linear unit (denoted as 'relu' in Fig. 2). For each target value X^D at time t, the features in the past five-time steps, i.e., $t-4, t-3, t-2, t-1, t$, are provided as the inputs of the network. To achieve a better performance, both of the inputs and target data are scaled to $[0,1]$. The mean square error is used as the loss function of the network. By minimizing the loss function of the network outputs, the predictive model for $X^D(t)$ can be obtained.

2) Modelling for $X^S(t)$

Since the output power of PVs are highly affected by the solar irradiation and weather conditions, there exist strong stochastic fluctuations in the real PV output power. Similarly, human activities influence the load power, resulting in violent power deviations. In order to describe such stochastic characters mathematically, Wiener process is introduced in [6], [12], [21] for the modelling of load power. For literatures regarding the applications of stochastic differential equations in power modelling of PVs, readers can consult [6], and the references therein.

Within the branch of stochastic analysis, there exist many types of stochastic differential equations, e.g., stochastic differential equations driven by Wiener process, stochastic differential equations driven by Lévy process [28], etc. It is notable that stochastic differential equations driven by Ornstein-Uhlenbeck process [24] has been considered in [12] for the power modelling of loads. Similar to the modelling techniques introduced in [12], the stochastic part $X^S(t)$ is assumed to be an Ornstein-Uhlenbeck process. The expression of $X^S(t)$ is given as follows,

$$\mathrm{d}X^S(t) = k\big(\mu - X^S(t)\big)\mathrm{d}t + \sigma \mathrm{d}w(t), \tag{2}$$

where $w(t)$ is a Weiner process; k, μ and σ are constant parameters. Given the real values of $X(t)$, as long as the predictive model for $X^D(t)$ is obtained, the values of $X^S(t)$ can be calculated immediately. Based on $X^S(t)$ series, the unknown parameters in (2) could be estimated with maximize likelihood estimation methods [29].

Supposing there are $n+1$ consecutive values of $X^S(t)$ and the time step between two consecutive values is h. According to [24], the transition density for X_{ih}^S and $X_{(i-1)h}^S$ is

$$X_{ih}^S | X_{(i-1)h}^S \sim N\left(\mu(1-e^{-kh}) + e^{-kh} X_{(i-1)h}^S, \sigma^2(1-e^{-2kh})/(2k)\right). \tag{3}$$

The Euler discrete approximation is

$$X_{ih}^S = k\mu h + (1-kh) X_{(i-1)h}^S + \sigma N(0,h), \tag{4}$$

where $N(0,h)$ stands for a normal distribution. Suppose we have n pairs of X_{ih}^S and $X_{(i-1)h}^S$, the logarithm likelihood function is

$$H(k,\mu,\sigma) = -\frac{1}{2h\sigma^2}\sum_{i=1}^n \left[X_{ih}^S - k\mu h + (kh-1)X_{(i-1)h}^S\right]^2 - n\ln(\sigma) - \frac{n}{2}\ln(2\pi h) \tag{5}$$

By solving (6),

$$\begin{cases} \frac{\partial H(k,\mu,\sigma)}{\partial k} = 0, \\ \frac{\partial H(k,\mu,\sigma)}{\partial \mu} = 0, \\ \frac{\partial H(k,\mu,\sigma)}{\partial \sigma} = 0, \end{cases} \tag{6}$$

the recursive formulas of the parameters are obtained in (7),

$$\begin{cases} k = -\frac{1}{h}\frac{\sum_{i=1}^n \left(X_{(i-1)h}^S - \mu\right)\left(X_{ih}^S - X_{(i-1)h}^S\right)}{\sum_{i=1}^n \left(X_{(i-1)h}^S - \mu\right)^2}, \\ \mu = \frac{1}{khn}\sum_{i=1}^n \left(X_{ih}^S + (kh-1)X_{(i-1)h}^S\right), \\ \sigma = \frac{1}{hn}\sum_{i=1}^n \left(X_{ih}^S - X_{(i-1)h}^S + kh\left(X_{(i-1)h}^S - \mu\right)\right)^2. \end{cases} \tag{7}$$

Since (7) is still difficult to be solved directly, the parameters can be obtained via expectation maximization algorithm [30].

C. PV Power Modelling

As is described in Section II-B, the power model of PVs is formulated as

$$\begin{cases} P_{PV}(t) = P_{PV}^D(t) + P_{solar}(t)P_{PV}^S(t), \\ \mathrm{d}P_{PV}^S(t) = k_{PV}\left(\mu_{PV} - P_{PV}^S(t)\right)\mathrm{d}t + \sigma_{PV}\mathrm{d}w_{PV}(t), \end{cases} \tag{8}$$

where $P_{PV}^D(t)$ is predicted with the NNs introduced in Fig. 2; $P_{solar}(t)$ is the theoretical solar irradiation power; $w_{PV}(t)$ is a scaler Weiner process defined in a given complete filtered probability space $(\Omega, \mathcal{F}, \mathcal{P}; \mathcal{F}_t)$.

To obtain a predictive model for $P_{PV}^D(t)$, the smoothed PV power data is fed as the target values of the network outputs. The feature vector used as the network input is composed of weather data and theoretical solar irradiation power. The whole year data is divided into training set and test set randomly. The former one is used for training, and the latter one is used for testing. The performance of the trained model is shown in Table I.

TABLE I
THE MEAN SQUARE ERROR OF THE MODEL FOR $P_{PV}^D(t)$.

Data Set	Mean Square Error
Training Set	0.0072
Test Set	0.0060

Once we obtain the model of $P_{PV}^D(t)$, the undetermined parameters in (8) can be estimated with (7).

D. Load Power Modelling

Similar with the PV power modelling approach, the power of loads can be formulated with

$$\begin{cases} P_L(t) = P_L^D(t) + P_L^S(t), \\ \mathrm{d}P_L^S(t) = k_L\left(\mu_L - P_L^S(t)\right)\mathrm{d}t + \sigma_L \mathrm{d}w_L(t), \end{cases} \tag{9}$$

where $P_L^D(t)$ is described with the model introduced in Fig. 2; $w_L(t)$ is a scaler Weiner process defined in a given complete filtered probability space $(\Omega, \mathcal{F}, \mathcal{P}; \mathcal{F}_t)$. We assume that $w_{PV}(t)$ and $w_L(t)$ are independent. For the $P_L^D(t)$ model training, weather data, hour of day, day of week and day of year are used as the input feature vectors. The training process and results are shown in Table III.

TABLE III
THE MEAN SQUARE ERROR OF THE MODEL FOR $P_L^D(t)$.

Data Set	Mean Square Error
Training Set	0.0030
Test Set	0.0018

The parameters for $P_L^S(t)$ are estimated after the model is trained. Finally, we obtain $k_L = 1.1372$, $\mu_L = 8.9251$, and $\sigma_L = 61.8400$.

According to the results illustrated in Table I and Table III, the prediction performance of the trained network appears to be satisfactory, and no obvious over-fitting is occurred.

E. The Power Modelling of Other Components in MG

1) Modelling for output power of MTs

Similar with the modelling approach for MT output power introduced in [6], [14], we apply the following linear ordinary differential equation to describe the power dynamics of MTs,

$$dP_{MT}(t) = -\frac{1}{T_{MT}}\left(P_{MT}(t) - K_{MT}u_{MT}(t)\right)dt, \tag{10}$$

where K_{MT} is the control signal gain (the maximized MT power); $u_{MT}(t)$ is the control input signal.

2) Modelling for output power of BES devices

We denote SOC of the BES devices at time t as $SOC(t)$. Similar with [31], the following ordinary differential equation is used to describe the SOC dynamics,

$$\mathrm{d}SOC(t) = \frac{\eta(P_{BES}(t))P_{BES}(t)}{Q_S}\mathrm{d}t, \tag{11}$$

where the coefficient $\eta\big(P_{BES}(t)\big)$ in (11) is defined as follows,

$$\eta\big(P_{BES}(t)\big) \triangleq \begin{cases} \eta_{in}\ , & P_{BES}(t) \geq 0, \\ 1/\eta_{out}, & P_{BES}(t) < 0. \end{cases} \tag{12}$$

Let us denote the lower bound for SOC as C_{min} and the upper bound for SOC as C_{max}. A reasonable range of SOC is that $SOC(t) \in [C_{min}, C_{max}]$; see, e.g., [31], [32].

3) Modelling for output power of the ER

ERs are core devices in the field of EI. When the considered MG is lack of power, the ER shall be regulated to deliver power from other connected multi-MGs to the considered MG. If there is surplus power in the considered MG, and even the local BES devices are almost fully charged, the abundant power can be transmitted to other connected multi-MGs. When power supply-demand balance in the considered MG is achieved autonomously, the connected ER is not required to work. When all the ERs in a wide area are functioning, energy can be transmitted in the way like information exchange in Internet. Without ERs, the energy routing network cannot be realized [7]-[10].

In this paper, the studied MG is connected with the remote multi-MGs via an ER. We assume that the ER has three different working modes: transmitting power from the considered MG to the interconnected multi-MGs; stopped; transmitting power from the interconnected multi-MGs to the considered MG. We denote the controller for the ER as $u_{ER}(t)$ which takes values within $[-1,1]$. The power model of the ER is formulated by

$$P_{ER}(t) = K_{ER}u_{ER}(t), \tag{13}$$

where K_{ER} is the pre-set value for the power transmitted through the ER. When $u_{ER}(t) = 0$, the ER is stopped functioning; when $u_{ER}(t)$ takes any positive value within (0,1], the ER transmits power from the considered MG to the multi-MGs; when $u_{ER}(t)$ takes any negative value within $[-1,0)$, the ER transmits energy from the multi-MGs to the considered MG.

III. The Stochastic Optimal Control Problem Formulation and Solution

In this section, the control objectives are formulated as a stochastic optimal control problem which is solved via dynamic programming approach.

A. Synthetical System Modelling for the Considered MG

Based on the power dynamical expressions (8)-(13), we formulate the synthetical system for our considered MG in (14) and (15),

$$\begin{cases} P_{PV}(t) = P_{PV}^D(t) + P_{solar}(t)P_{PV}^S(t), \\ \mathrm{d}P_{PV}^S(t) = k_{PV}\big(\mu_{PV} - P_{PV}^S(t)\big)\mathrm{d}t + \sigma_{PV}\mathrm{d}w_{PV}(t), \\ P_L(t) = P_L^D(t) + P_L^S(t), \\ \mathrm{d}P_L^S(t) = k_L\big(\mu_L - P_L^S(t)\big)\mathrm{d}t + \sigma_L\mathrm{d}w_L(t), \\ dP_{MT}(t) = -\frac{1}{T_{MT}}\big(P_{MT}(t) - K_{MT}u_{MT}(t)\big)dt, \\ \mathrm{d}SOC(t) = \frac{\eta(P_{BES}(t))P_{BES}(t)}{Q_S}\mathrm{d}t, \\ P_{ER}(t) = K_{ER}u_{ER}(t). \end{cases} \tag{14}$$

According to the assumption that the power deviation on power bus could be transmitted to other MGs by ER or be absorbed by BES devices almost instantly, the relationship between power generation and consumption is formulated as

$$P_{MT}(t) + P_{PV}(t) - P_L(t) \pm P_{ER}(t) \pm P_{BES}(t) = 0. \tag{15}$$

Mathematically, for $t \in [0,T]$, the system defined with (14) and (15) can be rewritten into

$$\begin{cases} \mathrm{d}x(t) = [Ax(t) + Bu(t) + C]\mathrm{d}t + D\mathrm{d}W(t), \\ x(0) = x_0, \end{cases} \tag{16}$$

where $x(t) = [P_{PV}^S(t) \quad P_L^S(t) \quad P_{MT}(t) \quad SOC(t)]'$ is state variable, $u(t) = [u_{MT} \quad u_{ER}]'$ is the control input. In the diffusion term of (16), $W(t) = [w_{PV}(t) \quad w_L(t)]'$.

The coefficients A, B, C, D in (16) are presented as follows:

$$A = \begin{bmatrix} -k_{PV} & 0 & 0 & 0 \\ 0 & -k_L & 0 & 0 \\ 0 & 0 & -1/T_{MT} & 0 \\ a(x(t),t)P_{solar}(t) & -a(x(t),t) & a(x(t),t) & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \dfrac{K_{MT}}{T_{MT}} & 0 \\ 0 & K_{ER} \end{bmatrix}, \quad C = \begin{bmatrix} k_{PV}\mu_{PV} \\ k_L\mu_L \\ 0 \\ c(x(t),t) \end{bmatrix}, \quad D = \begin{bmatrix} \sigma_{PV} & 0 \\ 0 & \sigma_L \\ 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Besides, we have

$$a(x(t),t) = \frac{\eta\big(P_{MT}(t) + P_{PV}(t) - P_L(t) + P_{ER}(t)\big)}{Q_S},$$

and

$$c(x(t),t) = a(x(t),t)\big(P_{PV}^{D}(t) - P_{L}^{D}(t)\big).$$

B. Formulation of Cost Functional

In this paper, the main objective is to realize the bottom-up energy management principle in EI. In this subsection, a cost functional describing such control target is formulated mathematically.

As long as the autonomous power balance in the local MG is preferably to be achieved, power flow through the ER shall be kept within a relatively small amount. On the other hand, the cooperative functioning of MTs, BES devices and the ER is the premise of the bottom-up energy management approach. Irrational utilization of MTs, BES devices and the ER would lead to relatively high operation and maintenance costs. The costs brought by power regulation of MTs, power exchange via the ER and charging/discharging of BES devices shall be considered when formulating the cost functional.

Let us introduce the characteristic function

$$\mathbb{I}_{[C_{min},C_{max}]}SOC(t) = \begin{cases} 1, & \text{if } SOC(t) \in [C_{min}, C_{max}], \\ 0, & \text{if } SOC(t) \notin [C_{min}, C_{max}]. \end{cases} \quad (17)$$

Based on (17), the objective cost functional $J\big(0, x(0); u(\cdot)\big)$ is defined as follows, (time t omitted)

$$J\big(0, x(0); u(\cdot)\big) \triangleq \mathbb{E}\left[\int_0^T \left(\alpha_1 \frac{p(|P_{BES}(t)| - P_{BES}^{L})}{P_{BES}^{L}} + \alpha_2 u_{MT}^2 + \alpha_3 u_{ER}^2 \mathbb{I}_{[C_{min},C_{max}]}SOC(t) + \alpha_4\big[1 - \mathbb{I}_{[C_{min},C_{max}]}SOC(t)\big]\right) \mathrm{d}t\right], \quad (18)$$

where $\mathbb{E}$ stands for the mathematical expectation. In (18), constants α_1, α_2, α_3 and α_4 are system weighting coefficients. The formulation of the terms in (18) is introduced as follows.

Since the lifetime of BES devices is directly related to the depth of discharge [32], in order to achieve the rational utilization of BES devices, $P_{BES}(t)$ shall be restricted within the maximum power throughput P_{BES}^{L}. The term

$$\mathbb{E}\left[\int_0^T \frac{p(|P_{BES}(t)| - P_{BES}^{L})}{P_{BES}^{L}}\, dt\right]$$

in (18) corresponds to such regulation principle. Here, the penalty function $p(|P_{BES}(t)| - P_{BES}^{L})$ is defined as follows,

$$p(|P_{BES}(t)| - P_{BES}^{L}) \triangleq \begin{cases} |P_{BES}(t)| - P_{BES}^{L}, & |P_{BES}(t)| > P_{BES}^{L}, \\ 0, & |P_{BES}(t)| \le P_{BES}^{L}. \end{cases}$$

When the input/output power of BES devices is within the upper bound P_{BES}^{L}, the term

$$\frac{p(|P_{BES}(t)| - P_{BES}^{L})}{P_{BES}^{L}}$$

equals to zero. If the input/output power of BES devices exceeds the upper bound, this term will change from zero to a positive value $|P_{BES}(t)| - P_{BES}^{L}$ (increasing with the excessive amount). Thus, the controller that results in irrational utilization of BES devices will be punished by such penalty function. In this sense, the service life of BES devices can be prolonged effectively.

Although strong controllers in MTs can achieve satisfactory controlling effect, the situation of over-control might happen, which brings extra equipment maintenance cost. Setting the term

$$\mathbb{E}\left[\int_0^T \alpha_2 u_{MT}^2\, dt\right]$$

in the cost functional (18) and minimizing (18) can effectively avoid the situation of over-control in MTs.

The key to realize the bottom-up energy management principle in EI is to control the value of the term

$$\mathbb{E}\left[\int_0^T \big(\alpha_3 u_{ER}^2 \mathbb{I}_{[C_{min},C_{max}]}SOC(t) + \alpha_4[1 - \mathbb{I}_{[C_{min},C_{max}]}SOC(t)]\big)\, dt\right] \quad (19)$$

to a relatively low level. When $SOC(t) \in [C_{min}, C_{max}]$, $t \in [0,T]$, according to (17), we have $\mathbb{I}_{[C_{min},C_{max}]}SOC(t) = 1$, and the integrand of (19) becomes

$$\alpha_3 u_{ER}^2 \mathbb{I}_{[C_{min},C_{max}]}SOC(t).$$

Since the bottom-up principle addresses realizing autonomous local power balance with priority, power transmitted via the ER shall be minimized under such circumstances, which is reflected in setting the term

$$\mathbb{E}\left[\int_0^T \left(\alpha_3 u_{ER}^2 \mathbb{I}_{[C_{min},C_{max}]}SOC(t)\right) dt\right]$$

in the cost functional (18).

At any time $t \in [0,T]$, if the local BES devices are almost fully charged and more energy is consistently being generated in the considered MG, (alternatively, if the local BES devices are almost exhausted and more power demand is consistently required), i.e., $SOC(t) \notin [C_{min}, C_{max}]$, then energy routing strategy shall be implemented by the ER. Under such circumstance, according to (17), we have

$$\mathbb{I}_{[C_{min},C_{max}]}SOC(t) = 0,$$

and the integrand in (19) becomes

$$\alpha_4[1 - \mathbb{I}_{[C_{min},C_{max}]}SOC(t)].$$

In this sense, the energy flow via the ER is not required to be restricted. However, the status of $SOC(t) \notin [C_{min}, C_{max}]$ is not recommended in MG operation, which means that the value of

$$\mathbb{E}\left[\int_0^T \left(\alpha_4[1 - \mathbb{I}_{[C_{min},C_{max}]}SOC(t)]\right) dt\right]$$

is required to be regulated within a relatively low level.

In (18), the size and weight of system coefficients α_1, α_2, α_3 and α_4 can be adjusted according to the actual engineering requirements. If the cost functional $J\big(0, x(0); u(\cdot)\big)$ in (18) is minimized by the optimal controller $u(\cdot)$, our control target is achieved.

C. Solution to the Stochastic Optimal Control Problem

Our target is to find a controller such that the cost functional $J\big(0, x(0); u(\cdot)\big)$ in (18) is minimized subject to (s.t.) system (16), i.e.,

$$\begin{aligned} &\min_{u(\cdot)\in\mathcal{U}} \qquad\qquad J\big(0, x(0); u(\cdot)\big), \\ &s.t. \quad dx(t) = (Ax(t) + Bu(t) + C)dt + DdW(t), \end{aligned} \tag{20}$$

where $x(0) = x_0$; $\mathcal{U}$ is the set of all admissible controllers. An admissible controller $u(\cdot)$ for the stochastic system (16) is a $\mathcal{F}_t$-adapted process under which (16) has a unique solution.

The stochastic optimal control problem (20) can be solved with BOCOPHJB toolbox [25]. The main results are demonstrated in Section IV.

IV. Numerical Simulations

A. Validation for the Hybrid Models for Power of PVs and loads

The data on August 17th 2016 in [22] is chosen for simulation. Firstly, the predictions of PV and load power are obtained. The comparisons of the real data and our proposed hybrid models for PV and load power are presented in Fig. 3 and Fig. 4, respectively. The real power data are drawn with solid lines and the output of the NNs are shown with dash dotted lines. The dashed line in Fig. 3 corresponds to the predicted PV power obtained from (8) in one simulation. The dashed line in Fig. 4 corresponds to the predicted load power from (9) in one simulation, similarly. Within 200 times of simulations, almost all of the output power of the proposed hybrid PV model locates in the grey region in Fig. 3. From Fig. 3, we can see that the real PV power curve is mostly located in the grey area. Similar results and comments apply to Fig. 4. Thereby, the effectiveness of our proposed hybrid modelling method has been demonstrated.

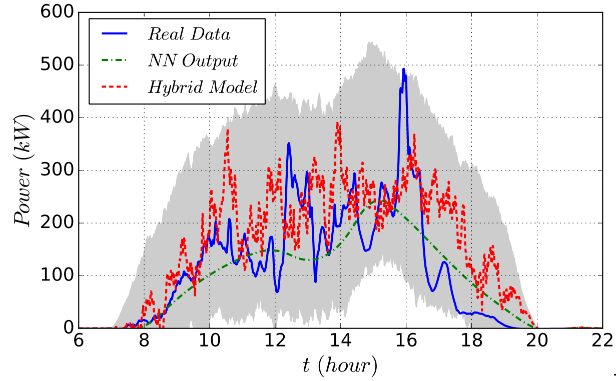

Fig. 3 Typical modelling result for PV power.

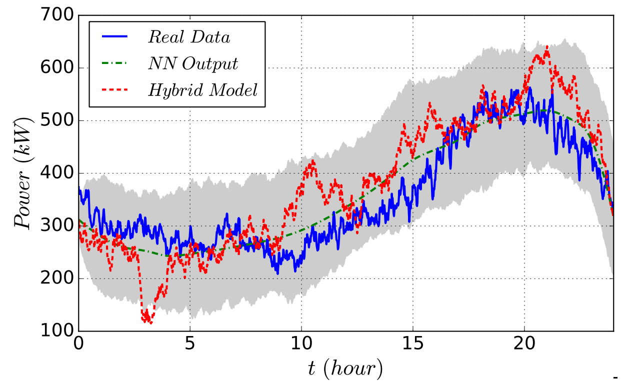

Fig. 4 Typical modelling result for load power.

B. Typical Simulation Results

The time interval for the simulation is set to be from 6 am to 4 pm, and the parameters are provided in Table V.

TABLE V
SYSTEM PARAMETERS

Parameters	Values	Parameters	Values
T_{MT}	$3\ min$	C_{Min}	0.2
K_{MT}	$400\ kW$	C_{Max}	0.9
K_{ER}	$100\ kW$	C_{Min}^{T}	0.5
P_{BES}^{L}	$200\ kW$	C_{Max}^{T}	0.8
Q_S	$125\ kWh$	η_{in}	0.97
α_1	3.5	η_{out}	0.95
α_2	4.9	α_3	10.7
α_4	1×10^{20}		

The power dynamics of loads, PVs and MTs are illustrated in Fig. 5. The power of BES devices is shown in Fig. 6. As is shown in Fig. 5, the output power of MTs responds properly to the deviations of the PV and load power. From Fig. 6, we see that the major parts of the BES power curve are restricted within 200kW.

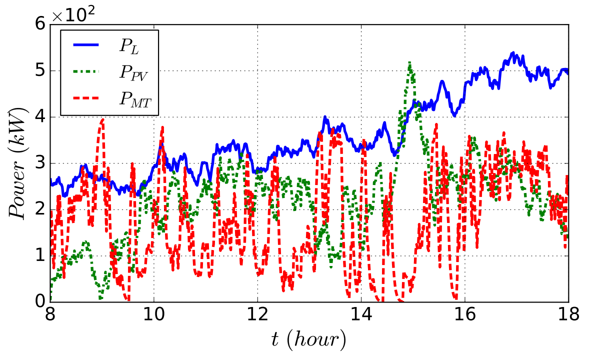

Fig. 5 Power dynamics of loads, PVs and MTs under the proposed control method.

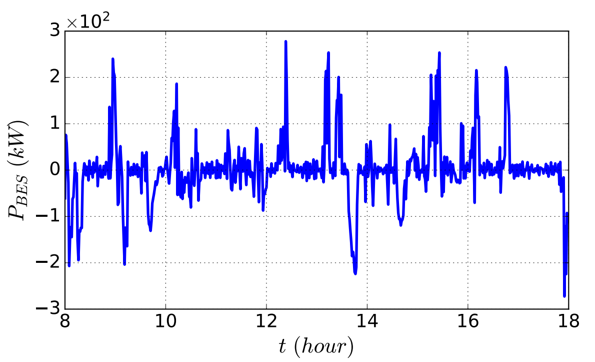

Fig. 6 Power dynamics of BES devices under the proposed control method.

In Fig. 7, the energy throughput curves of BES devices and that of the ER are presented. We see that the drastic power deviations on the power bus are mainly eliminated by BES devices. Due to the constraints for SOC, it is possible that the power transmitted via the ER would be sometimes greater than that absorbed by BES devices.

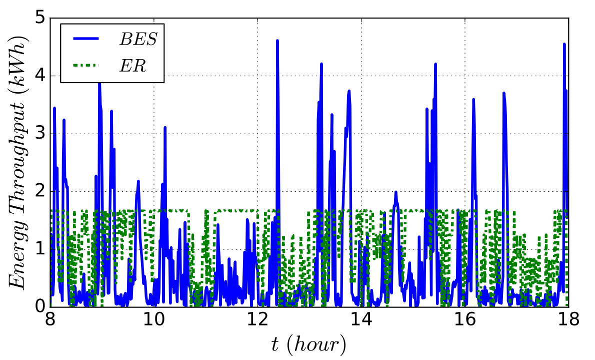

Fig. 7 Energy throughput of BES devices and the ER under the proposed control method.

The SOC curve is presented in Fig. 8. It is obvious that the SOC is kept within the constraints set in (17) successfully during the simulation period.

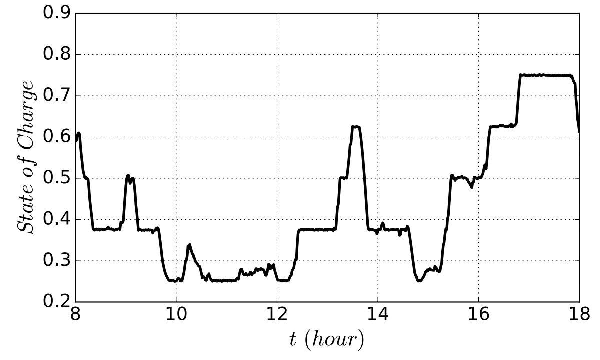

Fig. 8 SOC curve under the proposed control method.

To further demonstrate the advantages of introducing stochasticity into the system modelling, the performance of a deterministic control scheme is evaluated as follows.

By eliminating all the terms with randomness in (16), a deterministic system for the MG can be obtained. Thus, we are able to formulate a deterministic control problem similar to (20). The solutions to the deterministic systems are simulated as follows.

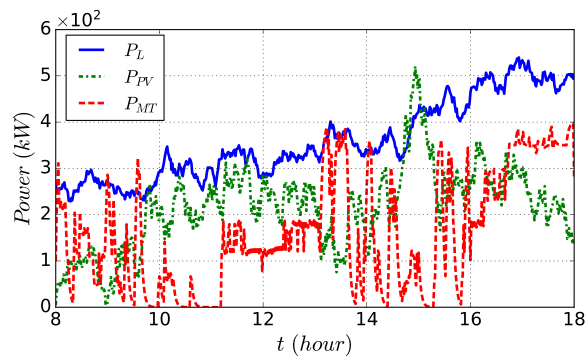

Fig. 9 Power dynamics of loads, PVs and MTs under the deterministic control scheme.

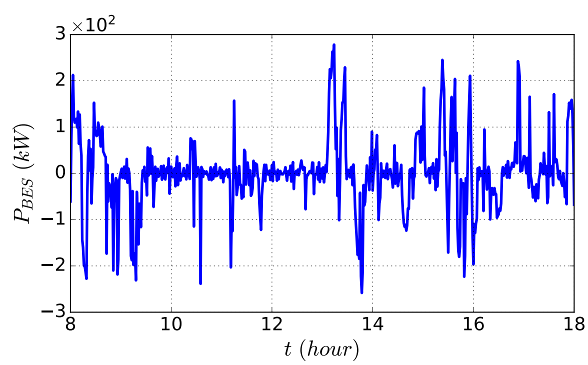

Fig. 10 Power dynamics of BES devices under the deterministic control scheme.

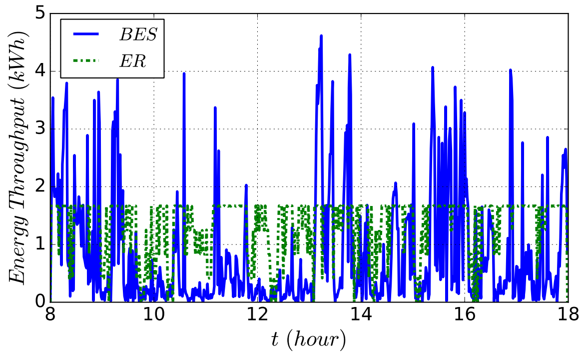

Fig. 11 Energy throughput of BES devices and the ER under the deterministic control scheme.

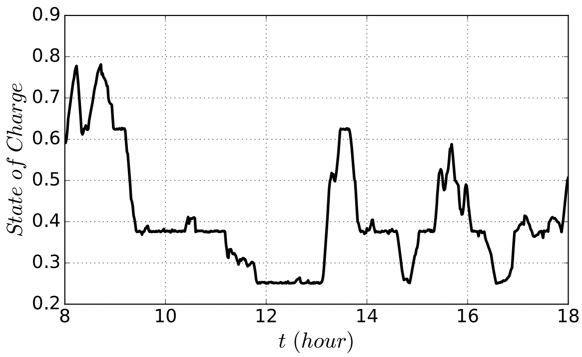

Fig. 12 SOC curve under the deterministic control scheme.

The power dynamics of loads, PVs and MTs are illustrated in Fig. 9. The power of BES devices is shown in Fig. 10. The energy throughput curves of BES devices and the ER are presented in Fig. 11. The SOC curve is shown in Fig. 12. From Fig. 6, Fig. 7, Fig. 10 and Fig. 11, we can see that the power throughput of BES devices under the proposed control method is smaller than that under the deterministic control scheme. Meanwhile, due to the neglect of the randomness in system (16), the deterministic controller is not able to respond to the stochastic power deviation timely in the considered MG system. Thus, under the deterministic control scheme, more fluctuations are appeared within the SOC curve in Fig. 12, especially during the time period [15,18], compared with the one in Fig. 8. With these simulation results, the advantages of our proposed hybrid modelling approach are presented.

The feasibility and effectiveness of our proposed control schemes are evaluated in this section. According to the above numerical results, our proposed stochastic control scheme ensures that the power deviations on the power bus are mainly absorbed within the considered MG, as well as realizing the rational utilization of BES devices, MTs and the ER. Based on these simulation results, the proposed optimal control scheme for the considered MG within an EI scenario is effective.

V. Conclusions

In summary, we formulate the bottom-up energy management issue in EI as a stochastic optimization problem. It is highlighted that a novel hybrid method for modelling the power of the PVs and loads is proposed using both RNNs and Ornstein-Uhlenbeck process simultaneously. By using the dynamic programming approach, the stochastic optimal control problem is solved numerically. The simulations demonstrate the usefulness of our proposed approach.

Since the algorithm proposed in [25] is a grid-based method, which requires a great deal of time and space to obtain the solution, designing reinforcement learning algorithms for the considered energy management problem will be the main direction for our future work.

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References

[1] M. Bhattacharya, S. R. Paramati, I. Ozturk, and S. Bhattacharya, "The effect of renewable energy consumption on economic growth: Evidence from top 38 countries," *Appl. Energy*, vol. 162, pp. 733-741, Jan. 2016.

[2] B. V. Mathiesen, H. Lund, D. Connolly, H. Wenzel, P. A. Østergaard, B. Möller, *et al.*, "Smart Energy Systems for coherent 100% renewable energy and transport solutions," *Appl. Energy*, vol. 145, pp. 139-154, May 2015.

[3] H. Han, X. Hou, J. Yang, J. Wu, M. Su, and J. M. Guerrero, "Review of power sharing control strategies for islanding operation of AC microgrids," *IEEE Trans. Smart Grid*, vol. 7, no. 1, pp. 200-215, Jan. 2016.

[4] A. T. Elsayed, A. A. Mohamed, and O. A. Mohammed, "DC microgrids and distribution systems: An overview," *Elect. Power Syst. Res.*, vol. 119, pp. 407-417, Feb. 2015.

[5] J. Rifkin, "The third industrial revolution: how lateral power is transforming energy, the economy, and the world," *Palgrave Macmillan*, New York, pp. 31–46, 2013.

[6] H. Hua, J. Cao, G. Yang, and G. Ren, "Voltage control for uncertain stochastic nonlinear system with application to energy Internet: Non-fragile robust H_∞ approach," *J. Math. Anal. Appl.*, vol. 463, no. 1, pp. 93-110, 2018.

[7] J. Cao, H. Hua, and G. Ren, "Energy use and the Internet," *The SAGE Encyclopedia of the Internet*, SAGE Publications, pp. 344-350, 2018.

[8] H. Hua, C. Hao, Y. Qin, and J. Cao, "A class of control strategies for energy Internet considering system robustness and operation cost optimization," *Energies*, vol. 11, no. 6, 1593, 2018.

[9] J. Miao, N. Zhang, C. Kang, J. Wang, Y. Wang, and Q. Xia, "Steady-state power flow model of energy router embedded AC network and its application in optimizing power system operation," *IEEE Trans. Smart Grid*, DOI 10.1109/TSG.2017.2672821.

[10] M. Gao, K. Wang, and L. He, "Probabilistic model checking and scheduling implementation of energy router system in energy Internet for green cities," *IEEE Trans. Ind. Inform.*, vol. 14, no. 4, pp. 1501-1510, Apr. 2018.

[11] A. Anvari-Moghaddam, J. M. Guerrero, J. C. Vasquez, and H. Monsef, "A Rahimi-Kian, Efficient energy management for a grid-tied residential microgrid," *IET Generation Transmission & Distribution*, vol. 11, no. 11, pp. 2752-2761, 2017.

[12] B. Heymann, J. F. Bonnans, F. Silva, and G. Jimenez, "A stochastic continuous time model for microgrid energy management," in *Proc. 2016 European Control Conf.*, Aalborg, Denmark, 2016, pp. 2084-2089.

[13] J. Zhao, and F. Dörfler, "Distributed control and optimization in DC microgrids," *Automatica*, vol. 61, pp. 18-26, Nov. 2015.

[14] H. Bevrani, M. R. Feizi, and S. Ataee, "Robust frequency control in an islanded microgrid: H_∞ and μ-synthesis approaches," *IEEE Trans. Smart Grid*, vol. 7, no. 2, Mar. 2016, pp. 706–717.

[15] Z. Wang, B. Chen, J. Wang, M. M. Begovic, and C. Chen, “Coordinated energy management of networked microgrids in distribution systems,” *IEEE Trans. Smart Grid*, vol. 6, no. 1, pp. 45–53, Jan. 2015.

[16] D. Tran and A. M. Khambadkone, “Energy management for lifetime extension of energy storage system in micro-grid applications,” *IEEE Trans. Smart Grid*, vol. 4, no. 3, pp. 1289-1296, Sept. 2013.

[17] A. K. Barnes, J. C. Balda, and A. Escobar-Mejía, “A semi-Markov model for control of energy storage in utility grids and microgrids with PV generation,” *IEEE Trans. Sust. Energy*, vol. 6, no. 2, pp. 546-556, Apr. 2015.

[18] D. L. Marino, K. Amarasinghe, and M. Manic, “Building energy load forecasting using deep neural networks,” in *Proc. 42nd Ann. Conf. IEEE Ind. Elect. Soc.*, Florence, Italy, Oct. 2016, pp. 7046-7051.

[19] M. Y. Zhai, “A new method for short-term load forecasting based on fractal interpretation and wavelet analysis,” *Int. J. Elec. Power & Energy Syst.*, vol. 69, pp. 241-245, Jul. 2015.

[20] H. Zhu, X. Li, Q. Sun, L. Nie, J. Yao, and G. Zhao, “A power prediction method for photovoltaic power plant based on wavelet decomposition and artificial neural networks,” *Energies*, vol. 9, no. 1, 11, 2015.

[21] F. Milano and R. Zárate-Miñano, “A systematic method to model power systems as stochastic differential algebraic equations,” *IEEE Trans. Power Syst.*, vol. 28, no. 4, pp. 4537-4544, 2013.

[22] “Dataport,” Pecan Street Inc., [Online]. Available: https://dataport.cloud/

[23] I. Sutskever, O. Vinyals, and Q. V. Le, “Sequence to sequence learning with neural networks,” *Advances in Neural Information Processing Syst.*, pp. 3104-3112, 2014.

[24] Q. Zang and L. Zhang, “Asymptotic behaviour of the trajectory fitting estimator for reflected Ornstein–Uhlenbeck processes,” *J. Theoretical Probability*, vol. 3, pp. 1-19, 2017.

[25] F. Bonnans, D. Giorgi, B. Heymann, P. Martinon and O. Tissot, “BocopHJB 1.0. 1--User Guide,” 2015.

[26] R. Carnegie, D. Gotham, D. Nderitu, and P. V. Preckel, “Utility scale energy storage systems,” State Utility Forecasting Group, Purdue University, vol. 1, 2013.

[27] H. Bevrani, B. François, and T. Ise, *Microgrid Dynamics and Control*, John Wiley & Sons, 2017.

[28] S. Albeverio, L. D. Persio, E. Mastrogiacomo, and B. Smii, “A Class of Lévy driven SDEs and their explicit invariant measures,” *Potential Anal.*, vol. 45, no. 2, pp. 229-259, Aug. 2016.

[29] A. Amiri, M. R. Maleki, and F. Sogandi, “Estimating the time of a step change in the multivariate-attribute process mean using ANN and MLE,” *Int. J. Data Analy. Tech. & Strategies*, vol. 10, no. 1, pp. 81-89, 2018.

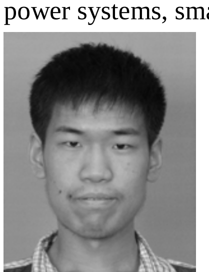
[30] R. Gupta, K. Audhkhasi, Z. Jacokes, A. Rozga, and S. Narayanan, “Modeling multiple time series annotations as noisy distortions of the ground truth: an expectation-maximization approach,” *IEEE Trans. Affective Computing*, vol. 9, no. 1, pp. 76-89, 2018.

[31] B. Heymann, J. F. Bonnans, P. Martinon, F. J. Silva, F. Lanas, and G. Jimenez-Estevez, “Continuous optimal control approaches to microgrid energy management,” *Energy Syst.*, pp. 1-19, 2015.

[32] D. Tran and A. M. Khambadkone, “Energy management for lifetime extension of energy storage system in micro-grid applications,” *IEEE Trans. Smart Grid*, vol. 4, no. 3, pp. 1289-1296, Sept. 2013.

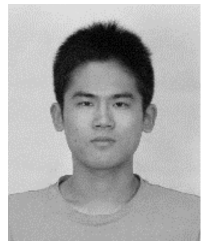
Haochen Hua (M’16) was born in Jiangsu, P. R. China in 1988. He received the B.Sc. degree in mathematics with finance in 2011, and the Ph.D. degree in mathematical sciences in 2016, both from the University of Liverpool, Liverpool, UK.

He is currently a post-doctoral researcher in the Research Institute of Information Technology, Tsinghua University, Beijing, P. R. China. His current research interests include optimal and robust control theory and its applications in power systems, smart grids, and the energy internet.



Yuchao Qin was born in Henan, P. R. China in 1994. He received the B.Sc. degree in automation in 2017 from Tsinghua University, Beijing, P. R. China.

He is currently a postgraduate student in the Research Institute of Information Technology, Tsinghua University, Beijing, P. R. China. His current research interests include control and optimization, machine learning and their applications in power system.



Chuantong Hao was born in Tianjin, P. R. China in 1994. He received the B.Sc. degree in 2016 from Tsinghua University, Beijing, P.R. China.

He is currently a postgraduate research student in the Research Institute of Information Technology, Tsinghua University, Beijing, P. R. China. His current research interests include control and optimization of the energy Internet and the design of energy routers.

Junwei Cao (SM’05) received his Ph.D. in computer science from the University of Warwick, Coventry, UK, in 2001. He received his bachelor and master degrees in control theories and engineering in 1998 and 1996, respectively, both from Tsinghua University, Beijing, China. He is currently Professor and Vice Dean of Research Institute of Information Technology, Tsinghua University, Beijing, China. He is also Director of Open Platform and Technology Division, Tsinghua National Laboratory for Information Science and Technology.

Prior to joining Tsinghua University in 2006, he was a Research Scientist at MIT LIGO Laboratory and NEC Laboratories Europe for about 5 years. He has published over 200 papers and cited by international scholars for over 18,000 times. He has authored or edited 8 books. His research is focused on distributed computing technologies and energy/power applications.

Prof. Cao is a Senior Member of the IEEE Computer Society and a Member of the ACM and CCF.